COMPUTER SYSTEMS AND ORGANIZATION

Part 1

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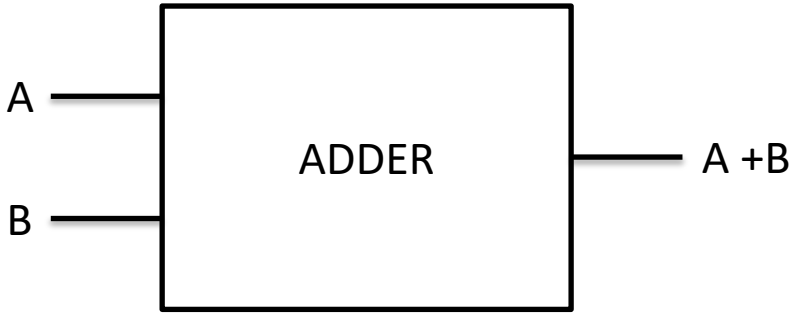
REVIEW

THE IDEA



THE CHALLENGE

Our gates only support 0 and 1s.

How can we represent other decimal numbers?

How can we present negative numbers?

What about fractions ☺?

DECIMAL

- Decimal numbers

1000's column
100's column
10's column
1's column

$$5374_{10} = 5 \times 10^3 + 3 \times 10^2 + 7 \times 10^1 + 4 \times 10^0$$

five thousands
three hundreds
seven tens
four ones

BINARY

8's column
4's column
2's column
1's column

$$1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 13_{10}$$

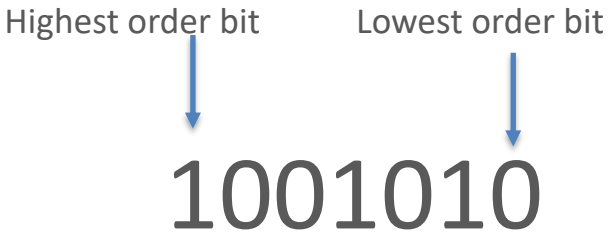
one eight
one four
no two
one one

BINARY CONVERSION EXAMPLES

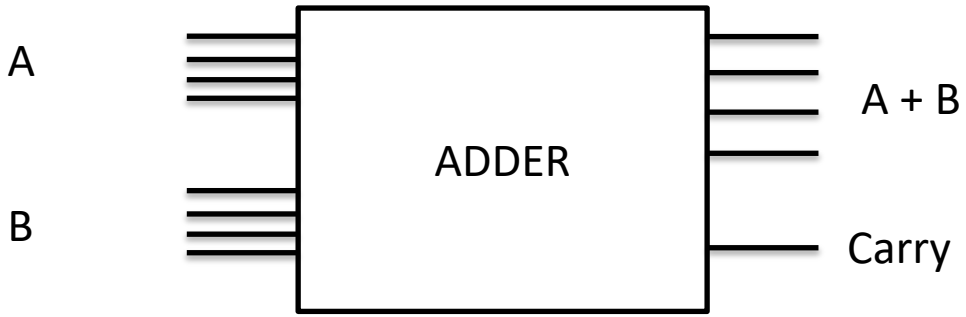
Convert 74 to binary = 1001010

74 ÷ 2 = 37 remainder 0
37 ÷ 2 = 18 remainder 1
18 ÷ 2 = 9 remainder 0
9 ÷ 2 = 4 remainder 1
4 ÷ 2 = 2 remainder 0
2 ÷ 2 = 1 remainder 0
1 ÷ 2 = 1 remainder 1

Convert 10101 to Decimal



4-BIT ADDER

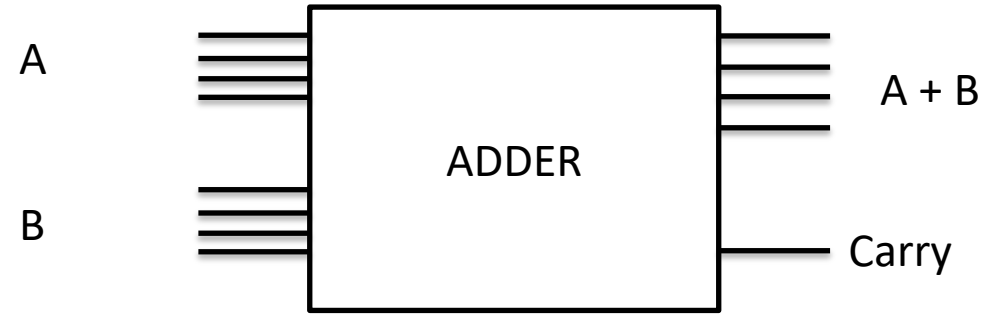


INPUTS AND OUTPUT OF OUR ADDER

What would the input be if wanted to add 5 and 9?

Notice we need to pick and order for the wires. More on this later ☺

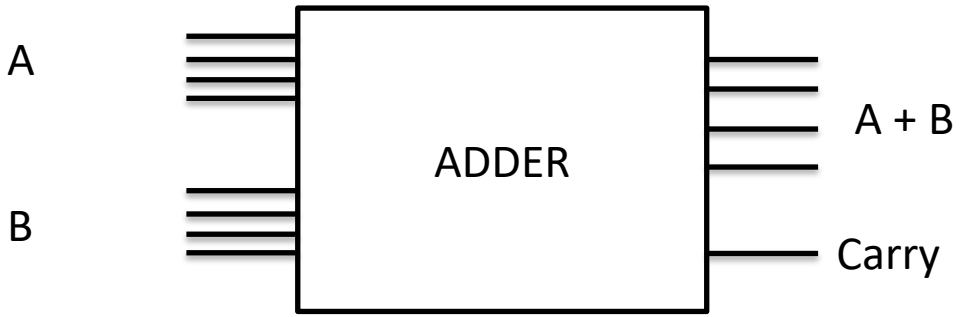
Which output lights would we want to light Up?



INPUTS AND OUTPUT OF OUR ADDER

What if we now added 7 and 9?

What would our inputs be, and which lights do we expect to light up?



ADDING

```
1 1 1 1   ← Carries
  0 1 1 1
+ 1 0 1 1
  0 0 1 0
```

Let's start by building a half adder something that just adds two bits.

Let's build a truth table.

A	B	A + B	C.out
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

We can implement
A + B with an XOR gate
And the C.out (Carry out)
With an AND gate

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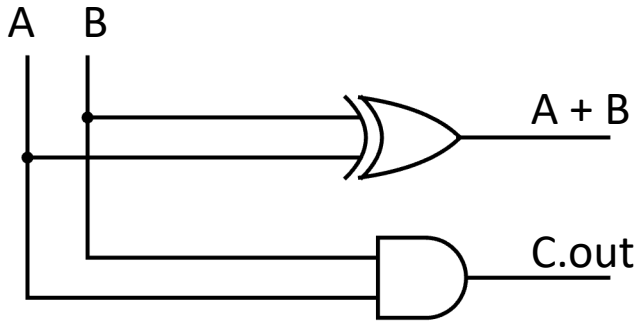
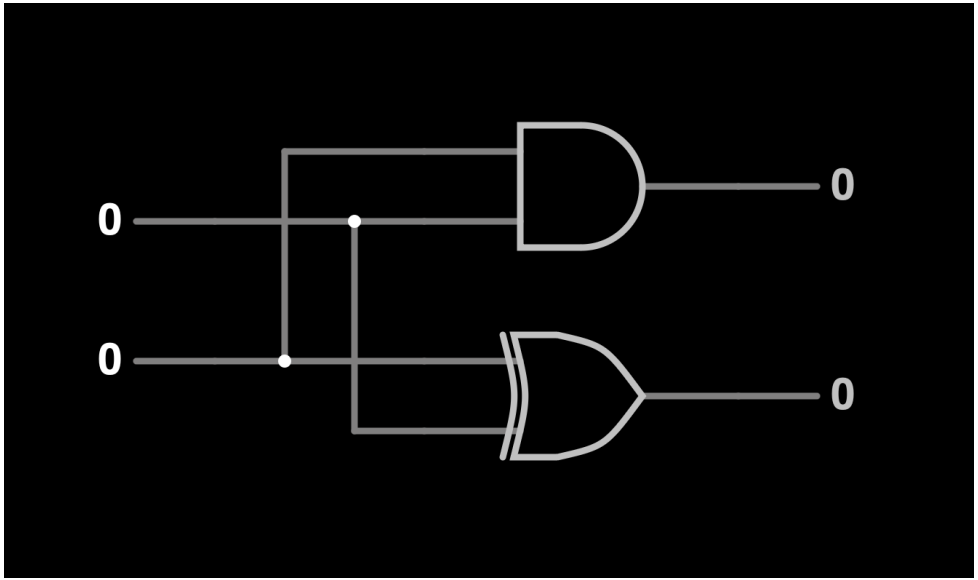


HALF ADDER DEMO



https://tinyurl.com/ygpea8v4

http://www.falstad.com/circuit/circuitjs.html?ctz=CQAgjCAMB0l3BWc0FwCwCY0HYEA4cEMElURTJyBTAWjDACgwE0QMs21KBmANj06VKGKOSZl2rMGlZ8B01sNEIGAGXAZ5vSnkphtbUQDMAhgBsAzlXJQ1GgZJC62HEZVOXrSSAwDu9lykDRx9-fWEOcIDQ8AMwTUDov1iI1kcQ5PitPQBOESiYsDyU8GLiXlswsoQK9JrK0vzgyIMfAFkQOXAZEDR9brS2FAYOrqxKPtquQwxhoA

ADDING

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1 1 1 1   ← Carries
  0 1 1 1
+ 1 0 1 1
  0 0 1 0
```

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A + B with an XOR gate
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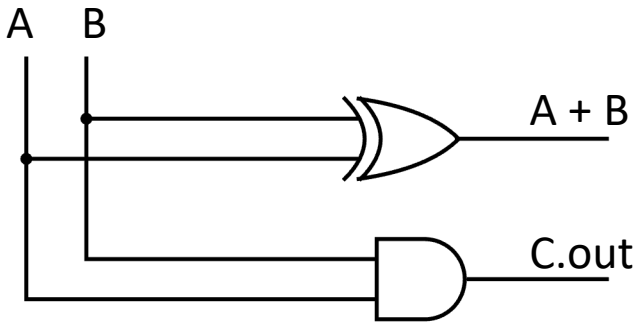
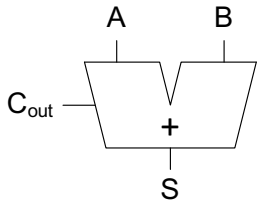


Half Adder

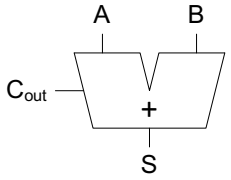


A	B	C_{out}	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$S = A \oplus B$

$C_{out} = AB$

Half Adder

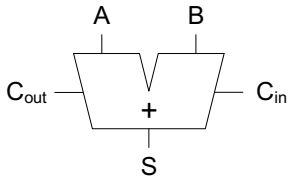


A	B	C_{out}	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

$S = A \oplus B$

$C_{out} = AB$

Full Adder



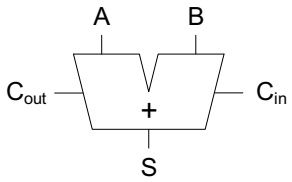
C_{in}	A	B	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$S = A \oplus B \oplus C_{in}$

$C_{out} = AB + AC_{in} + BC_{in}$

Note on special case 3 input xor.
Draw the three gates. Really
Two xors stacked.

Full Adder

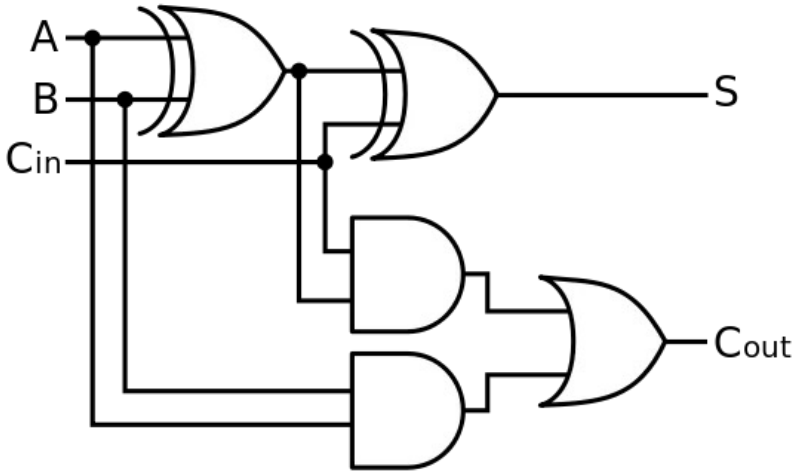


C_{in}	A	B	C_{out}	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

$S = A \oplus B \oplus C_{in}$

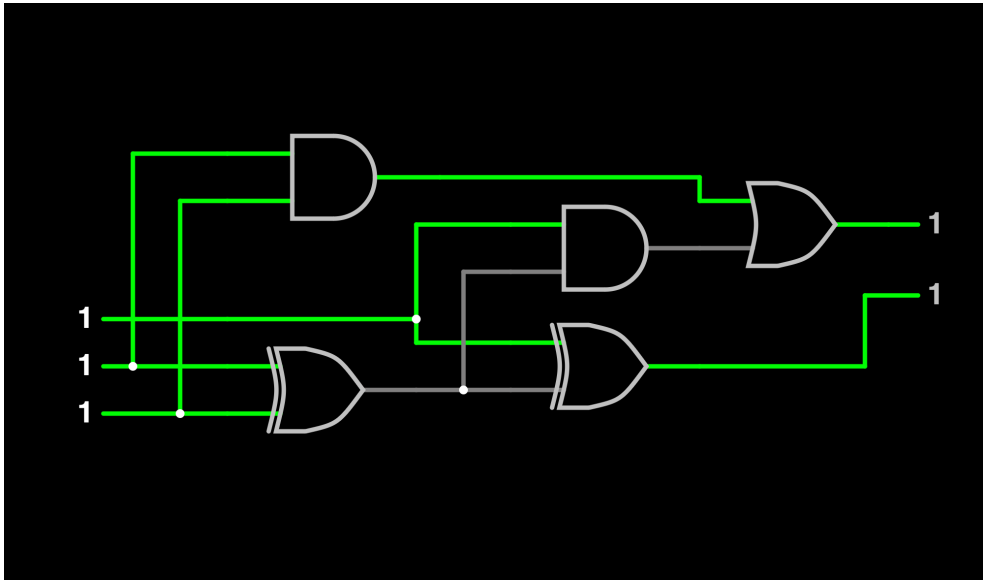
$C_{out} = AB + AC_{in} + BC_{in}$

```
1 1 1 1   ← Carries
  0 1 1 1
+ 1 0 1 1
  1 1 1 0
```



C.out has been rewritten to reduce the number of gates needed.

DEMO FULL ADDER

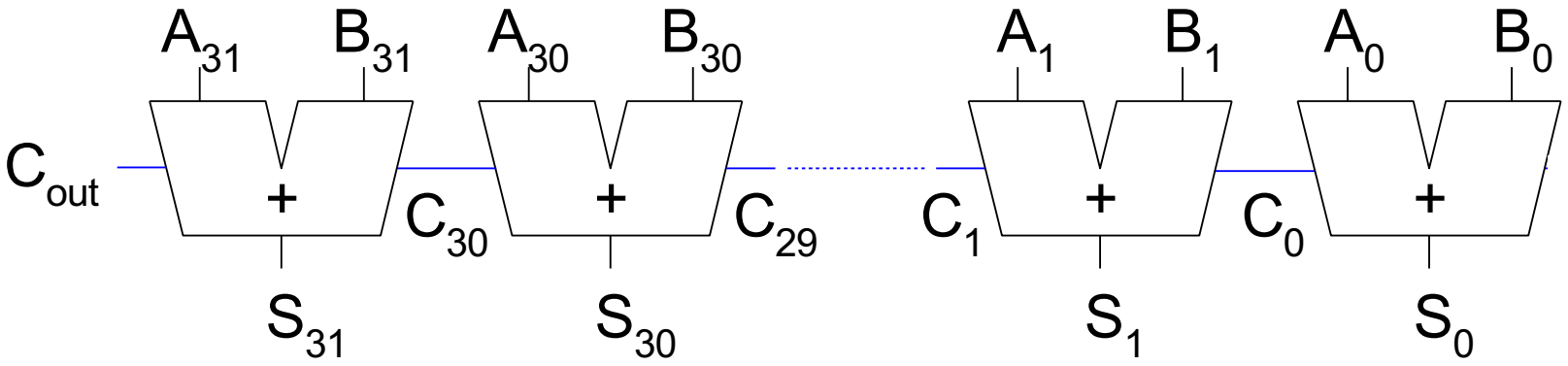


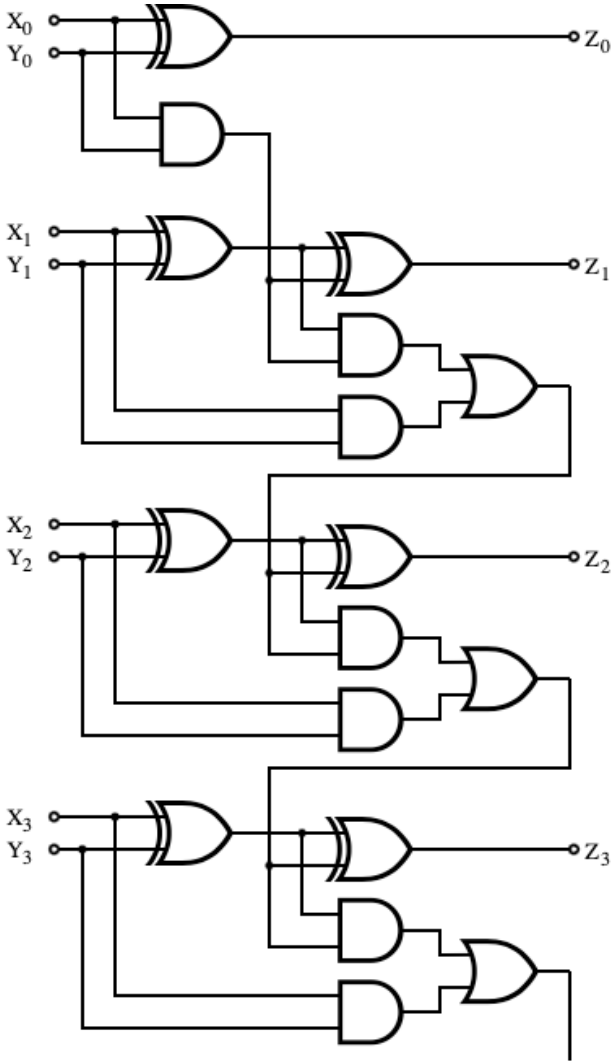
https://tinyurl.com/2cfbbshs

http://www.falstad.com/circuit/circuitjs.html?ctz=CQAgjCAMB0l3BWc0FwCwCY0HYEA4cEMElURTJyBTAWjDACgwE1w1WNsMRPveoeAhExYgAzGIBsPBNLSSOsgd1IMA7jy6buYyBy2R14vdvFTThjROlhJlfrcqWz0rPa1uoR-p59YvGn6sYOw8-s4OdqYY4d54eOAYCfLBSV4AsuQAnNyOIJL+eXwoRvR88WwcFc4hwWDceJRlASCN4PWt9krObcQ2oX1eADKdMnIJg7kgAGYAhgA2AM5U5MPt5ckTFVNzSytIhiO1YaxoE-47C8urNaHHzcc1HfcdSQnOKYnJCpUt1uA5YzBQHOXTAnQmfgfH6DBDYVzdIxwhHSZE8SDvBiZNEYDH5C544rCZhNO5pN5fZSrYlkf5geEgT706T2IQibhMqIIQF5VmqIA

RIPPLE CARRY ADDER

Next let's build a full adder





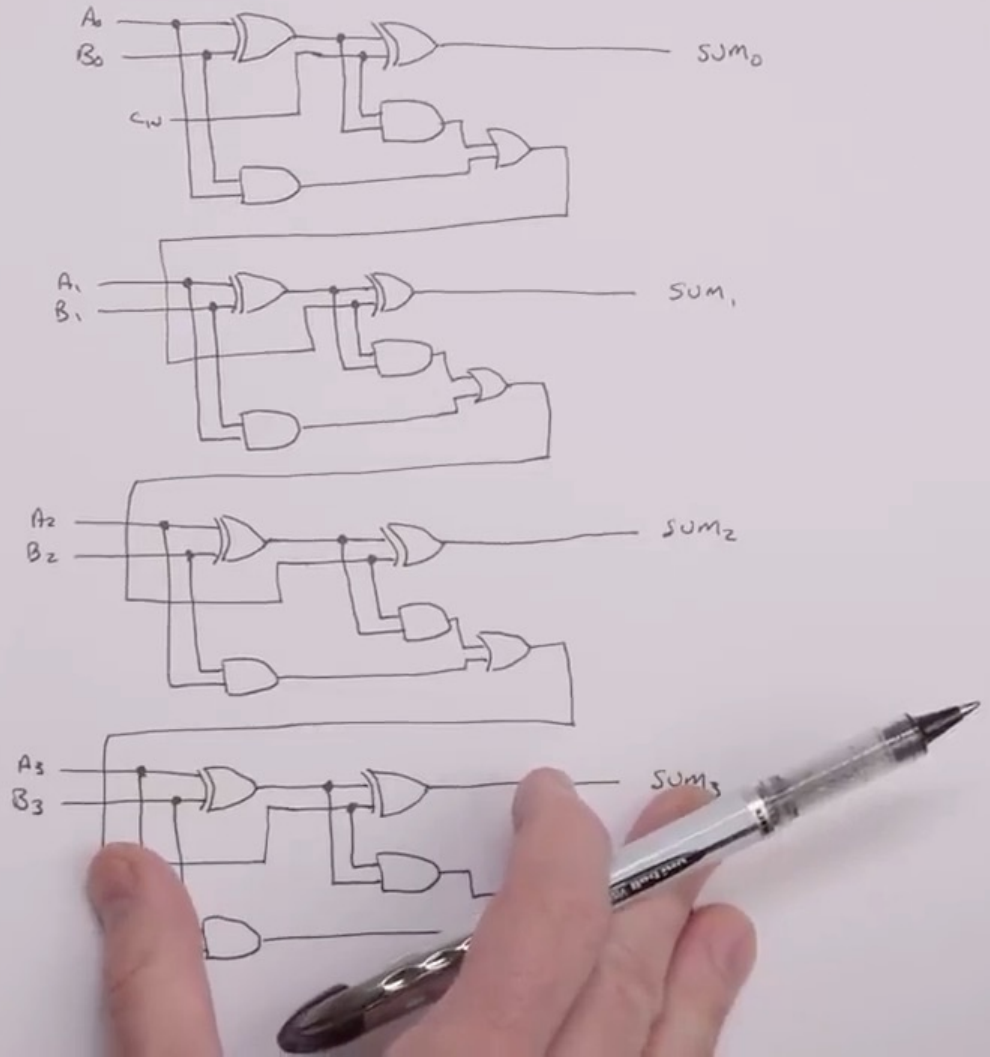
RIPPLE CARRY ADDER

1 1 1 1 ← Carries

0 1 1 1

+ 1 0 1 1

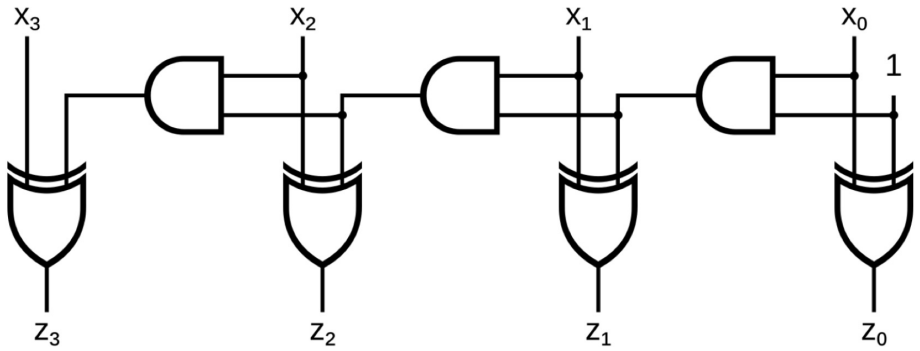
0 0 1 0


A0
B0
CW
SUM0
A1
B1
SUM1
A2
B2
SUM2
A3
B3
SUM3

CHALLENGE CAN WE BUILD A CIRCUIT THAT INCREMENTS A NUMBER BY ONE

Page 5: Circuits

10. [16 points] In class, we discussed a 4-bit increment circuit below that added 1 to the input.



How can we change this circuit to instead increment by 2, i.e., `x += 2`? Draw the new circuit below. *Note: you should not use more gates than the original circuit.*